ECL 4340

POWER SYSTEMS

LECTURE 9

TRANSMISSION LINE MODELS

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1

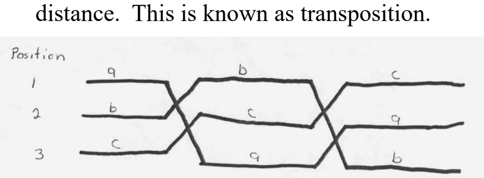
ANNOUNCEMENTS

- Read Chapter 5.
- Homework #5 is due Friday, October 7.

2

TRANSPOSITION

- To keep system balanced, over the length of a transmission line the conductors are rotated so each phase occupies each position on tower for an equal distance. This is known as transposition.



Aerial or side view of conductor positions over the length of the transmission line.

3

3

Inductance of Transposed Line

Define the geometric mean distance (GMD)

$$D_m \triangleq (d_{12} d_{13} d_{23})^{1/3}$$

Then for a balanced 3ϕ system $(I_a = -I_b - I_c)$

$$\lambda_a = \frac{\mu_0}{2\pi}\left[I_a \ln\frac{1}{r'} - I_a \ln\frac{1}{D_m}\right] = \frac{\mu_0}{2\pi} I_a \ln\frac{D_m}{r'}$$

Hence

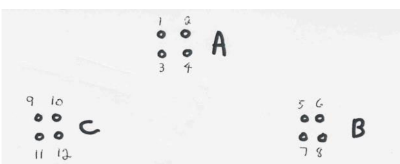
$$L_a = \frac{\mu_0}{2\pi}\ln\frac{D_m}{r'} = 2\times 10^{-7}\ln\frac{D_m}{r'} \text{ H/m}$$

4

4

Conductor Bundling

To increase the capacity of high voltage transmission lines, it is very common to use a number of conductors per phase. This is known as conductor bundling. Typical values are two conductors for 345 kV lines, three for 500 kV and four for 765 kV.



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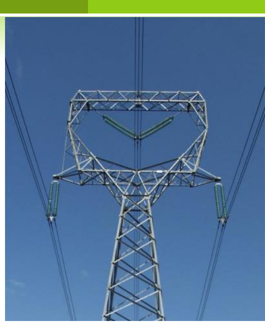
Bundled Conductor Pictures



The AEP Wyoming-Jackson Ferry 765 kV line uses 6-bundle conductors. Conductors in a bundle are at the same voltage!

Photo Source: BPA and American Electric Power

6

6

Inductance of Lines with Bundled Conductors

- The per phase inductance is

$$L_a = \frac{\mu_0}{2\pi} \ln\left(\frac{D}{R_b}\right)$$

where

$R_b \triangleq$ geometric mean radius (GMR) of bundle

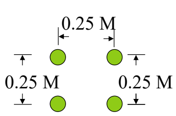
$= (r' d_{12} \ldots d_{1b})^{1/b}$ in general

When calculating the per phase resistance of bundled lines, the total resistance is R per conductor divided by b, where b is the number of conductors in the bundle

7

Bundle Inductance Example

Consider the previous example of the three phases symmetrically spaced 5 meters apart using wire with a radius of r = 1.24 cm. Except now assume each phase has 4 conductors in a square bundle, spaced 0.25 meters apart. What is the new inductance per meter?



$$r = 1.24 \times 10^{-2} \text{ m} \quad r' = 9.67 \times 10^{-3} \text{ m}$$

$$R_b = \left(9.67 \times 10^{-3} \times 0.25 \times 0.25 \times \sqrt{2} \times 0.25\right)^{1/4}$$

$$= 0.12 \text{ m (ten times bigger!)}$$

$$L_a = \frac{\mu_0}{2\pi} \ln \frac{5}{0.12} = 7.46 \times 10^{-7} \text{ H/m}$$

8

8

Inductance Example

- Calculate the per phase inductance and reactance of a balanced 3ϕ, 60 Hz, line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1cm radius.

Answer: D_m = 12.6 m, R_b = 0.0889 m
Inductance = 9.9 x 10^{-7} H/m, Reactance = 0.6 Ω/Mile

9

9

Line Capacitance

- High voltage transmission lines and cables can have significant capacitance

For the case of uniformly transposed lines we use the same D_m and a similar GMR as with inducance,

$$C = \frac{2\pi\varepsilon}{\ln \frac{D_m}{R_b^c}}$$

where $D_m = [d_{ab} d_{ac} d_{bc}]^{1/3}$

$R_b^c = (r d_{12} \cdots d_{1n})^{1/n}$ (note r NOT r')

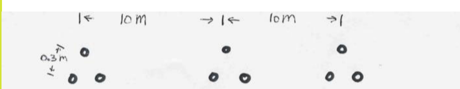
ε in air $= \varepsilon_o = 8.854 \times 10^{-12}$ F/m

10

10

Line Capacitance Example

- Calculate the per phase capacitance and susceptance of a balanced 3ϕ, 60 Hz, transmission line with horizontal phase spacing of 10m using three conductor bundling with a spacing between conductors in the bundle of 0.3m. Assume the line is uniformly transposed and the conductors have a 1cm radius.



11

11

Line Capacitance Example, cont'd

$$R_b^c = (0.01 \times 0.3 \times 0.3)^{1/3} = 0.0963 \text{ m}$$

$$D_m = (10 \times 10 \times 20)^{1/3} = 12.6 \text{ m}$$

$$C = \frac{2\pi \times 8.854 \times 10^{-12}}{\ln \frac{12.6}{0.0963}} = 1.141 \times 10^{-11} \text{F/m}$$

$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi 60 \times 1.141 \times 10^{-11} \text{F/m}}$$

$$= 2.33 \times 10^{8} \ \Omega\text{-m (NOT } \Omega\text{/m)}$$

12

12

Additional Transmission Topics

- **Multi-circuit lines**: Multiple lines often share a common transmission right-of-way. This DOES cause mutual inductance and capacitance, but is often ignored in system analysis.
- **Cables:** There are about 3000 miles of underground ac cables in U.S. Cables are primarily used in urban areas. In a cable the conductors are tightly spaced, (< 1ft) with oil impregnated paper commonly used to provide insulation
  - inductance is lower
  - capacitance is higher, greatly limiting cable length

13

13

Additional Transmission Topics

- **Ground wires:** Transmission lines are usually protected from lightning strikes with a ground wire. This topmost wire (or wires) helps to attenuate the transient voltages/currents that arise during a lighting strike. The ground wire is typically grounded at each pole.
- **Corona discharge:** Due to high electric fields around lines, the air molecules become ionized. This causes a crackling sound and may cause the line to glow!

14

14

Additional Transmission Topics

- **Shunt conductance:** Usually ignored. A small current may flow through contaminants on insulators.
- **DC Transmission:** Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical for several specialized applications
  - long distance overhead power transfer (> 400 miles)
  - long cable power transfer such as underwater
  - providing an asynchronous means of joining different power systems (such as ERCOT to Eastern or Western grids)

15

15

Transmission Line Models

- Previous lectures have covered how to calculate the distributed inductance, capacitance and resistance of transmission lines.
- In this section we will use these distributed parameters to develop the transmission line models used in power system analysis.

16

16

Transmission Lines

Transmission Lines

Equivalent Circuits:

The short line approximation: for less than 80 km (50 miles)

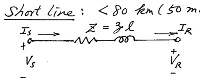
The Medium Line approximation: for 80~250 km (150 miles)

The Long Line approximation: for longer than 250 km

17

Transmission Lines

1. Short line: < 80 km (50 miles)



$z = R + j\omega L$ Ω/m

$Z = z\ell$

ℓ: length in [m]

V_S: line-to-neutral voltage.

Relation between the sending-end and the receiving-end variables:

$$V_S = V_R + Z I_R$$
$$I_S = I_R$$

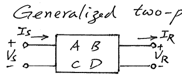
In a matrix form,

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

18

Transmission Lines

Generalized two-port network:



$V_s = A\,V_R + B\,I_R$

$I_s = C\,V_R + D\,I_R$

NOTE: For linear, passive, bilateral network,

$AD - BC = 1$

For symmetric network,

$A = D$

Inverse relationship:

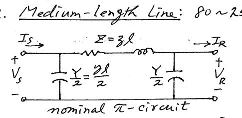
Since the determinant of the (A, B, C, D) matrix is $AD - BC = 1$, we have

$$\begin{bmatrix} V_R \\ I_R \end{bmatrix} = \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

19

Transmission Lines

2. Medium-length Line: 80 ~ 250 km (50 ~ 150 miles)



nominal π-circuit

Shunt admittance:

$y = G + j\omega C$ S/m

$Y = yl$

$$V_s = V_R + Z\left(I_R + \frac{Y}{2}V_R\right) = \left(1 + \frac{ZY}{2}\right)V_R + Z I_R$$

$$\begin{aligned} I_s &= I_R + \frac{Y}{2}V_R + \frac{Y}{2}V_s \\ &= I_R + \frac{Y}{2}V_R + \frac{Y}{2}\left(1 + \frac{ZY}{2}\right)V_R + \frac{Y}{2}Z I_R \\ &= Y\left(1 + \frac{ZY}{4}\right)V_R + \left(1 + \frac{ZY}{2}\right)I_R \end{aligned}$$

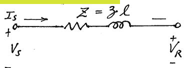
$$\therefore \begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 + \frac{ZY}{2} & Z \\ Y\left(1 + \frac{ZY}{4}\right) & 1 + \frac{ZY}{2} \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

20

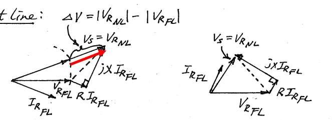
Transmission Lines

3. Voltage Regulation:

$$\% VR = \frac{|V_{RNL}| - |V_{RFL}|}{|V_{RFL}|} \times 100$$



Short line: $\Delta V = |V_{RNL}| - |V_{RFL}|$



(a) lagging p.f.
% VR is positive

(b) leading p.f.
% VR is negative

21

Transmission Lines

Medium line:

$$V_S = A V_R + B I_R, \quad I_R = 0 \text{ for No Load}$$

$$\Rightarrow \quad V_S = A V_{RNL}$$

$$\text{or} \quad V_{RNL} = \frac{V_S}{A} \quad (= V_S \text{ for short line})$$

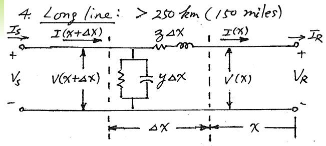
For EHV lines $\pm 5\%$ of rated voltage, i.e., 10 % VR is acceptable.

Line loadability is determined by
- Thermal limit: for short line
- Voltage-drop limit: for longer line upto 300 km
- Steady-state stability limit: over 300 km

22

Transmission Lines

4. Long line: > 250 km (150 miles)



z: series impedance Ω/m
y: shunt admittance S/m

For a small section of length Δx,

$$V(x+\Delta x) = V(x) + (z\Delta x)\, I(x)$$
$$I(x+\Delta x) = I(x) + (y\Delta x)\, V(x)$$

$$\Rightarrow \quad \Delta V = V(x+\Delta x) - V(x) = (z\Delta x)\, I(x)$$
$$\Delta I = I(x+\Delta x) - I(x) = (y\Delta x)\, V(x)$$

23

Transmission Lines

$$\Rightarrow \quad \Delta V = V(x+\Delta x) - V(x) = (z\Delta x)\, I(x)$$
$$\Delta I = I(x+\Delta x) - I(x) = (y\Delta x)\, V(x)$$

As $\Delta x \to dx$,

$$dV = I\, z\, dx \qquad dI = V\, y\, dy$$

$$\text{or} \quad \frac{dV}{dx} = z\, I(x) \qquad \frac{dI}{dx} = y\, V(x) \qquad (1)$$

Differentiating with respect to x,

$$\frac{d^2V}{dx^2} = z\,\frac{dI}{dx} \qquad \frac{d^2I}{dx^2} = y\,\frac{dV}{dx}$$

$$\text{or} \quad \frac{d^2V}{dx^2} = zy\, V(x) \qquad \frac{d^2I}{dx^2} = zy\, I(x)$$

: These are wave equations

Solution is in the form of

$$V(x) = A\, e^{\sqrt{zy}\, x} + B\, e^{-\sqrt{zy}\, x}$$

24

Transmission Lines

$\frac{d^2V}{dx^2} = zy\,V(x)$ $\quad$ $\frac{d^2I}{dx^2} = zy\,I(x)$

Define the propagation constant γ as

$$\gamma = \sqrt{yz} = \alpha + j\beta$$

where

α = the attenuation constant

β = the phase constant

Use the Laplace Transform to solve.

$$\frac{d^2V}{dx^2} = \gamma^2 V \qquad s^2V(s) = \gamma^2 V(s)$$

System has a characteristic equation

$$(s^2 - \gamma^2) = (s-\gamma)(s+\gamma) = 0$$

25

Transmission Lines

System has a characteristic equation

$$(s^2 - \gamma^2) = (s-\gamma)(s+\gamma) = 0$$

$\Rightarrow s = \pm\gamma$ or $e^{\pm\gamma x}$ in the spatial domain

Solution is in the form of

$$V(x) = A\,e^{\sqrt{zy}\,x} + B\,e^{-\sqrt{zy}\,x}$$

Note: $\frac{dV}{dx} = \sqrt{zy}\,(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x})$ (2)

$$\frac{d^2V}{dx^2} = zy\,(A\,e^{\sqrt{zy}\,x} + B\,e^{-\sqrt{zy}\,x})$$

$= zy\,V(x)$, thus satisfying the wave equation.

26

Transmission Lines

Note: $\frac{dV}{dx} = \sqrt{zy}\,(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x})$ (2)

$$\frac{d^2V}{dx^2} = zy\,(A\,e^{\sqrt{zy}\,x} + B\,e^{-\sqrt{zy}\,x})$$

$= zy\,V(x)$, thus satisfying the wave equation.

$\frac{dV}{dx} = z\,I(x)$ $\qquad$ $\frac{dI}{dx} = y\,V(x)$ (1)

Since $V(x)$ and $I(x)$ are coupled by (1), with (2),

$$I(x) = \frac{1}{z}\frac{dV}{dx}$$

$$= \frac{1}{\sqrt{\frac{z}{y}}}\,(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x})$$

27

Transmission Lines

Solution is in the form of

$$V(x) = A\,e^{\sqrt{zy}\,x} + B\,e^{-\sqrt{zy}\,x}$$

$$I(x) = \frac{1}{z}\frac{dV}{dx} = \frac{1}{\sqrt{\frac{z}{y}}}\left(A\,e^{\sqrt{zy}\,x} - B\,e^{-\sqrt{zy}\,x}\right)$$

Define line parameters:

$\gamma = \sqrt{zy}$ $[m^{-1}]$, propagation constant

$= \alpha + j\beta$, α = attenuation constant

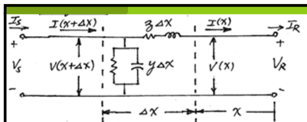
β = phase constant

$Z_c = \sqrt{\frac{z}{y}}$ $[\Omega]$, characteristic impedance

Then, the voltage and current at x are:

$$V(x) = A\,e^{\gamma x} + B\,e^{-\gamma x} \qquad (3)$$
$$I(x) = \frac{1}{Z_c}\left(A\,e^{\gamma x} - B\,e^{-\gamma x}\right)$$

28



Transmission Lines

Then, the voltage and current at x are:

$$V(x) = A\,e^{\gamma x} + B\,e^{-\gamma x} \qquad (3)$$
$$I(x) = \frac{1}{Z_c}\left(A\,e^{\gamma x} - B\,e^{-\gamma x}\right)$$

To find constants A and B, apply boundary conditions at $x = 0$:

$$V_R = V(0) = A + B$$
$$I_R = \frac{1}{Z_c}(A - B)$$

Solving for A & B,

$$A = \frac{V_R + Z_c I_R}{2} \qquad B = \frac{V_R - Z_c I_R}{2}$$

29

Transmission Lines

Then, the voltage and current at x are:

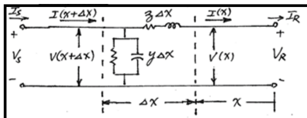
$$V(x) = A\,e^{\gamma x} + B\,e^{-\gamma x} \qquad (3)$$
$$I(x) = \frac{1}{Z_c}\left(A\,e^{\gamma x} - B\,e^{-\gamma x}\right)$$

$$A = \frac{V_R + Z_c I_R}{2} \qquad B = \frac{V_R - Z_c I_R}{2}$$

Substituting these in (3),

$$V(x) = \frac{V_R + Z_c I_R}{2}\,e^{\gamma x} + \frac{V_R - Z_c I_R}{2}\,e^{-\gamma x} \qquad (4)$$
$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2}\,e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2}\,e^{-\gamma x}}_{\text{reflected wave}}$$

30



Transmission Lines

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2} e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2} e^{-\gamma x}}_{\text{reflected wave}} \qquad (4)$$

NOTE: If $I_R = 0$ (open circuit, no load)

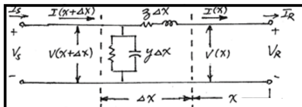
$$V(0)_{\text{incident}} = V(0)_{\text{reflected}} = \frac{V_R}{2}$$

$$I(0)_{\text{incident}} = -I(0)_{\text{reflected}} = \frac{V_R}{2Z_c}$$

$$\Rightarrow \quad V(0) = \frac{V_R}{2} + \frac{V_R}{2} = V_R$$

$$I(0) = \frac{V_R}{2Z_c} - \frac{V_R}{2Z_c} = 0$$

31



Transmission Lines

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2} e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2} e^{-\gamma x}}_{\text{reflected wave}} \qquad (4)$$

NOTE: If the line is terminated with characteristic impedance, Z_c, then

$$V_R = Z_c I_R$$

$\Rightarrow$ There is no reflected waves, which is called "Impedance Matching."

Impedance matching is common in communication systems. In power systems

$$Z_c = \sim 400\,\Omega \angle \sim 15^\circ \quad \text{for single circuit}$$
$$\sim 200\,\Omega \angle \sim 15^\circ \quad \text{for parallel circuit}$$

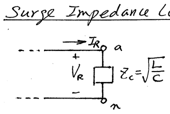
32

Transmission Lines

For loss less line (series resistance and shunt conductance are zero),

$$Z_c = \sqrt{\frac{z}{y}} = \sqrt{\frac{j\omega L}{j\omega C}} = \sqrt{\frac{L}{C}} \text{, surge impedance}$$

Surge Impedance Loading (SIL)



$$V_R = \frac{V_L}{\sqrt{3}}$$

$$I_R = \frac{V_R}{Z_c} = \frac{V_L}{\sqrt{3}\, Z_c}$$

$$SIL = \sqrt{3}\, V_L I_R = \frac{V_L^2}{Z_c}$$

$$= \frac{V^2_{\text{rated, line-to-line}}}{Z_c}$$

33

Transmission Lines

Wave Propagation:

$V(x)$ and $I(x)$ propagate (4) withe the propagation constant

$$\gamma = \sqrt{zy} = \alpha + j\beta$$

For loss-less line, $\alpha = 0$ and

$$\gamma = \sqrt{(j\omega L)(j\omega C)} = j\omega\sqrt{LC} = j\beta \quad [m^{-1}]$$

β: phase shift in rad/m.

A wave length is the distance required to change the phase of $V(x)$ or $I(x)$ by 2π radians or $360°$, i.e.,

$$\lambda\beta = 2\pi,$$

34

Transmission Lines

$\Rightarrow$ wave length, $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{1}{f\sqrt{LC}}$ [m]

$$= \frac{3\times 10^8}{60} = 5\times 10^6 \text{ m} = 5000 \text{ km} = 3100 \text{ mi}$$

Velocity of propagation:

$$v = f\cdot\lambda = \frac{1}{\sqrt{LC}} \quad [m/s]$$

$$= \frac{1}{\sqrt{\frac{\mu_0}{2\pi}\ln\frac{D}{r}\cdot\frac{2\pi\varepsilon_0}{\ln\frac{D}{r}}}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}} \simeq 3\times 10^8 \text{ m/s}$$

: speed of light in free space.

For cables, $\frac{\varepsilon}{\varepsilon_0} \simeq 3\sim 5$, implying that v is lower than that for overhead lines.

35

Transmission Lines

Long Line Model in Hyperbolic Form:

Recall that

$$\sinh\theta = \frac{e^{\theta} - e^{-\theta}}{2}, \quad \cosh\theta = \frac{e^{\theta} + e^{-\theta}}{2}$$

$$\left[\ \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}, \quad \cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}\ \right]$$

$$\Rightarrow \sinh\theta + \cosh\theta = e^{\theta}, \quad \cosh\theta - \sinh\theta = e^{-\theta}$$

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x}$$

$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2} e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2} e^{-\gamma x}}_{\text{reflected wave}} \quad (4)$$

36

TRANSMISSION LINES

$$V(x) = \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x} \quad (4)$$

$$I(x) = \underbrace{\frac{V_R/Z_c + I_R}{2} e^{\gamma x}}_{\text{incident wave}} - \underbrace{\frac{V_R/Z_c - I_R}{2} e^{-\gamma x}}_{\text{reflected wave}}$$

$$\sinh\theta + \cosh\theta = e^{\theta}, \quad \cosh\theta - \sinh\theta = e^{-\theta}$$

Thus, from (4),

$$\begin{aligned} V(x) &= \frac{V_R + Z_c I_R}{2} e^{\gamma x} + \frac{V_R - Z_c I_R}{2} e^{-\gamma x} \\ &= \frac{V_R + Z_c I_R}{2}(\sinh\gamma x + \cosh\gamma x) + \frac{V_R - Z_c I_R}{2}(\cosh\gamma x - \sinh\gamma x) \\ &= \cosh(\gamma x)\, V_R + Z_c \sinh(\gamma x)\, I_R \end{aligned}$$

$$\begin{aligned} I(x) &= \frac{V_R/Z_c + I_R}{2} e^{\gamma x} - \frac{V_R/Z_c - I_R}{2} e^{-\gamma x} \\ &= \frac{V_R/Z_c + I_R}{2}(\sinh\gamma x + \cosh\gamma x) - \frac{V_R/Z_c - I_R}{2}(\cosh\gamma x - \sinh\gamma x) \\ &= \frac{\sinh(\gamma x)}{Z_c} V_R + \cosh(\gamma x)\, I_R \end{aligned}$$

37

TRANSMISSION LINES

$$V(x) = \cosh(\gamma x)\, V_R + Z_c \sinh(\gamma x)\, I_R$$

$$I(x) = \frac{\sinh(\gamma x)}{Z_c} V_R + \cosh(\gamma x)\, I_R$$

$$\therefore \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} A(x) & B(x) \\ C(x) & D(x) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

where

$$\begin{aligned} A(x) = D(x) &= \cosh(\gamma x) \quad [pu] \\ B(x) &= Z_c \sinh(\gamma x) \quad [\Omega] \\ C(x) &= \frac{1}{Z_c}\sinh(\gamma x) \quad [S] \end{aligned}$$

Finally, at $x = \ell$, $V(\ell) = V_s$, $I(\ell) = I_s$, and

$$\begin{aligned} A = D &= \cosh(\gamma \ell) \quad [pu] \\ B &= Z_c \sinh(\gamma \ell) \quad [\Omega] \\ C &= \frac{1}{Z_c}\sinh(\gamma \ell) \quad [S] \end{aligned}$$

38

TRANSMISSION LINES

Computation of Hyperbolic Functions:

Recall that

$$\gamma = \sqrt{zy} = \alpha + j\beta \quad [m^{-1}], \text{ complex}$$

Thus, $e^{\gamma \ell} = e^{(\alpha + j\beta)\ell} = e^{\alpha \ell} e^{j\beta \ell} = e^{\alpha \ell} \angle \beta \ell$

Then,

$$\cosh(\gamma \ell) = \frac{e^{\gamma \ell} + e^{-\gamma \ell}}{2} = \frac{1}{2}\left(e^{\alpha \ell}\angle \beta \ell + e^{-\alpha \ell}\angle -\beta \ell\right)$$

$$\sinh(\gamma \ell) = \frac{e^{\gamma \ell} - e^{-\gamma \ell}}{2} = \frac{1}{2}\left(e^{\alpha \ell}\angle \beta \ell - e^{-\alpha \ell}\angle -\beta \ell\right)$$

39

Transmission Line Example

Assume we have a 765 kV transmission line with a receiving end voltage of 765 kV(line to line), a receiving end power $S_R = 2000 + j1000$ MVA and

$$z = 0.0201 + j0.535 = 0.535\angle 87.8° \ \Omega/\text{mile}$$

$$y = j7.75\times10^{-6} = 7.75\times10^{-6}\angle 90.0° \ ℧/\text{mile}$$

Then

$$\gamma = \sqrt{zy} = 2.036\ \angle 88.9° \ / \text{ mile}$$

$$Z_c = \sqrt{\frac{z}{y}} = 262.7\ \angle\text{-}1.1°\ \Omega$$

40

Transmission Line Example, cont'd

Do per-phase analysis, using single-phase power and line-to-neutral voltages.

Then at the receiving end,

$$V_R = 765/\sqrt{3} = 441.7\angle 0° \text{ kV}$$

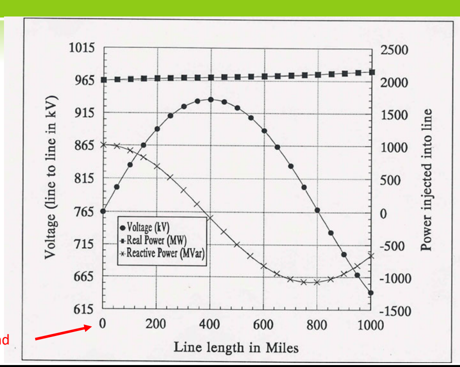
$$I_R = \left[\frac{(2000+j1000)\times 10^6}{3\times 441.7\angle 0°\times 10^3}\right]^* = 1688\angle -26.6° \text{ A}$$

$$V(x) = V_R\cosh(\gamma x) + I_R Z_c \sinh(\gamma x)$$

$$= 441{,}700\angle 0° \cosh(x\times 2.036\angle 88.9°) + 443{,}440\angle -27.7° \times \sinh(x\times 2.036\angle 88.9°)$$

41

Transmission Line Example, cont'd



Receiving end

42